Quantitative relations among causality measures with applications to nonlinear pulse-output network reconstruction

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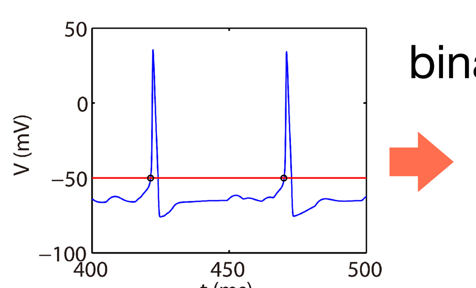
Motivations

- Inferring structural connectivity from nodes' activities.
- Results from different causal measures may be inconsistent.
- Causal (functional) connectivity generally inconsistent with structural connectivity.
- Some causal measures suffer curse of dimensionality when inferring network connectivities.
- Pulse-output signals are quite common in neural data.

Questions

- Relation between different causal measures.
- Relation between causality and structural connectivity.

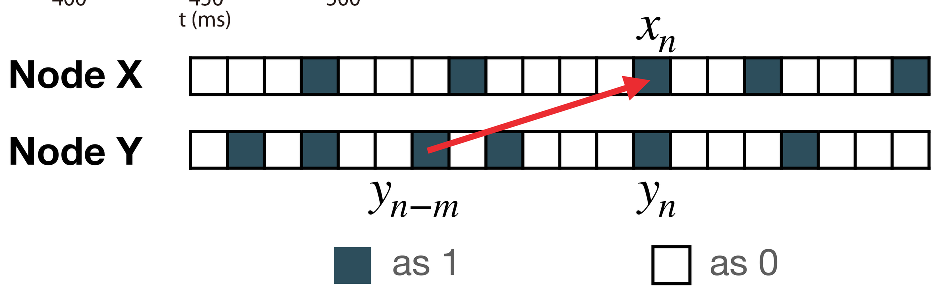
Pulse-output Signals

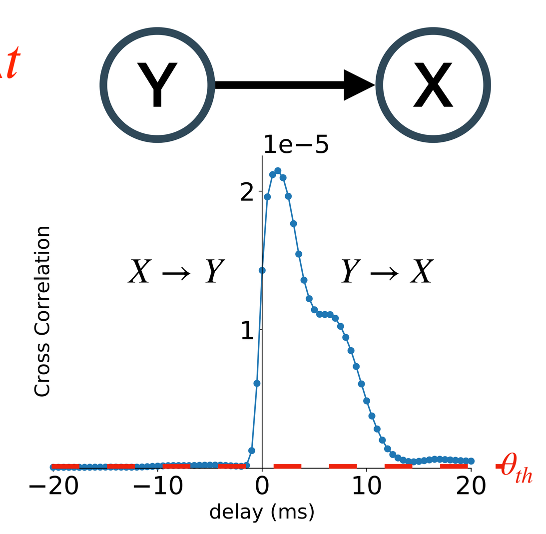


binary time series: time bin Δt

00000100000000010000

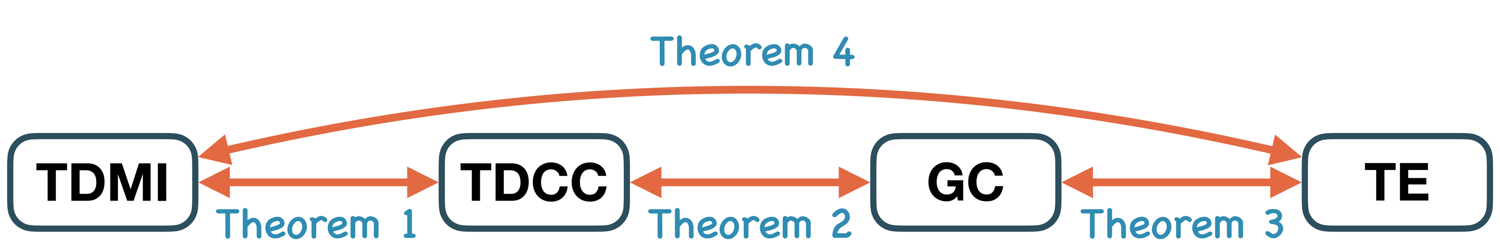
$$s_i(t) = \sum_l \delta(t - \tau_i^{(l)})$$





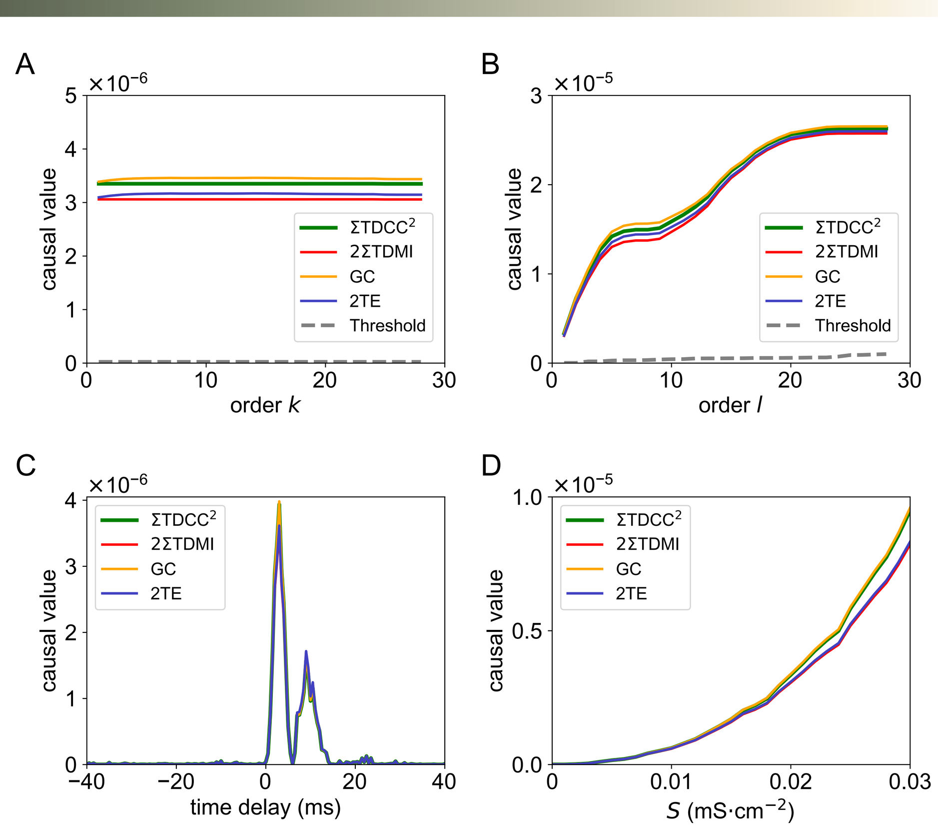
Theorems

- **Theorem 1:** $I(X,Y;m) = C^2(X,Y;m)/2 + O\left(\Delta t^2 \Delta p_m^3\right)$
- **Theorem 2:** $G_{Y\to X}(k,l;m) = \sum_{i=m}^{m+l-1} C^2(X,Y;i) + O\left(\Delta t^3 \Delta p_m^2\right)$
- **Theorem 3:** $T_{Y\to X}(k,l;m) = \sum_{i=m}^{m+l-1} I(X,Y;i) + O\left(\Delta t^3 \Delta p_m^2\right)$
- **Theorem 4:** $G_{Y\to X}(k,l;m) = 2T_{Y\to X}(k,l;m) + O(\Delta t^3 \Delta p_m^3)$

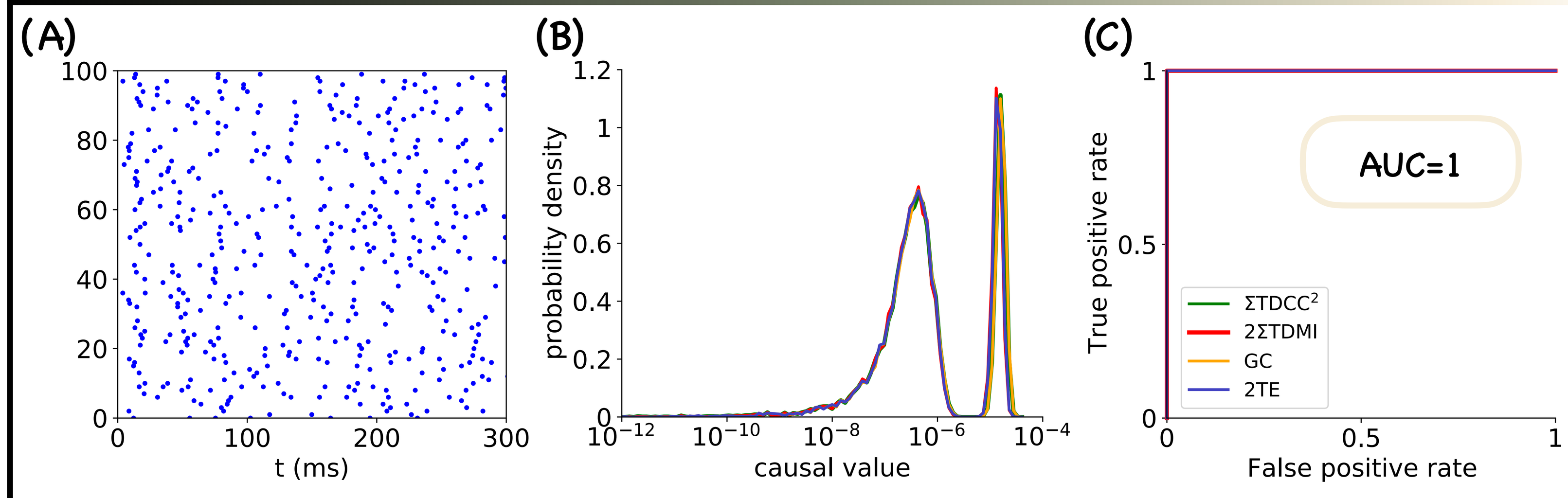


Numerical Verifications

- All four relations hold across k, l, m, and S.
- Causal values are insensitive to k.
- Causal values are significantly higher than threshold for all k and l.
- Causal values are quadratically related to S.



Reconstruction of HH Network



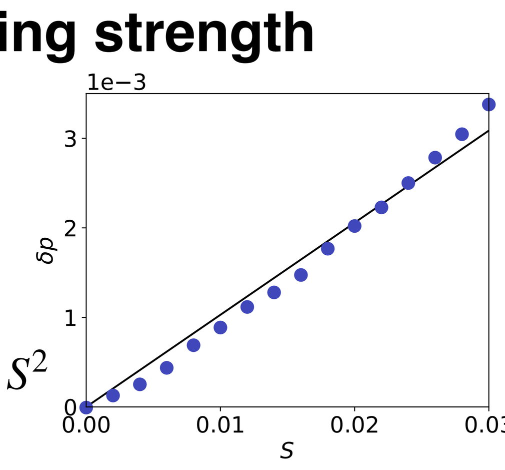
(A) Raster of 100-HH neurons. (B) Histogram of causal values. (C) ROC curves of reconstruction.

Direct connections: causality & coupling strength

$$\delta p_{Y\to X} = p\left(x_n = 1 \mid y_{n-m} = 1\right) - p\left(x_n = 1 \mid y_{n-m} = 0\right)$$

$$\delta p_{Y\to X} \propto S$$

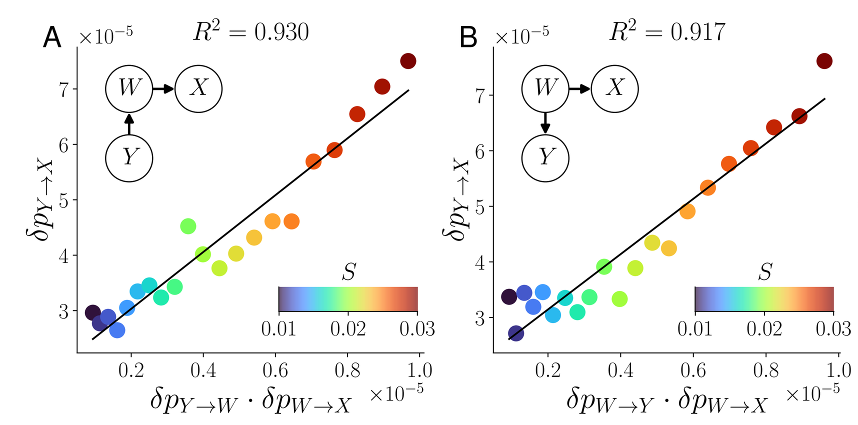
$$C(X,Y;m) = \delta p_{Y\to X}\sqrt{\frac{p_y - p_y^2}{p_x - p_x^2}} \propto S \quad TDMI, GC, TE \propto S^2$$



Indirect interactions

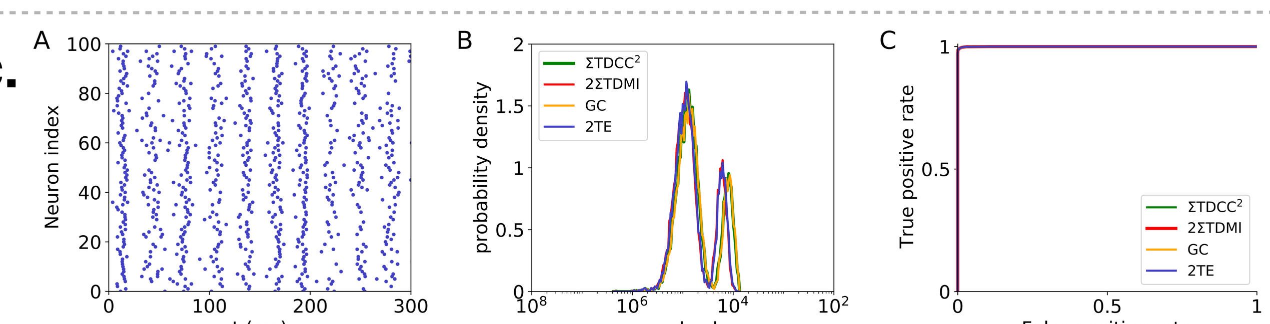
$$\delta p_{Y\to X} = O(\delta p_{Y\to W} \cdot \delta p_{W\to X})$$

$$\delta p_{Y\to X} = O(\delta p_{W\to Y} \cdot \delta p_{W\to X})$$



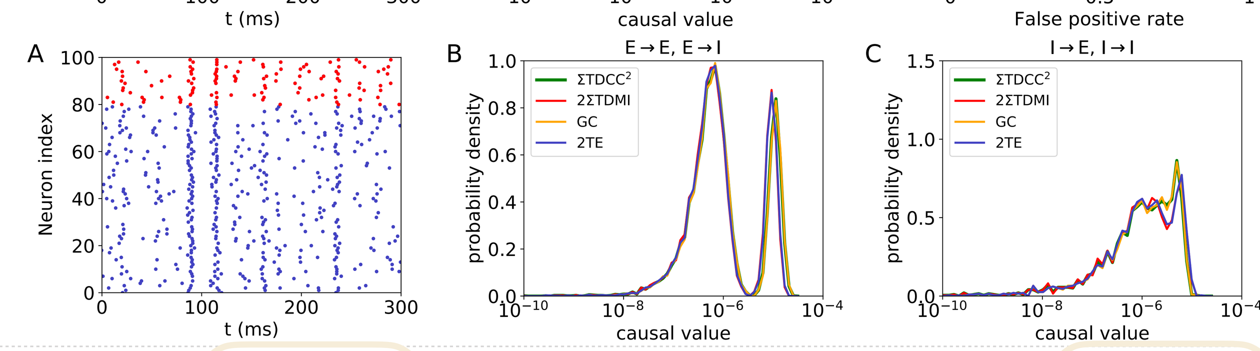
Near sync. state

AUC=0.99



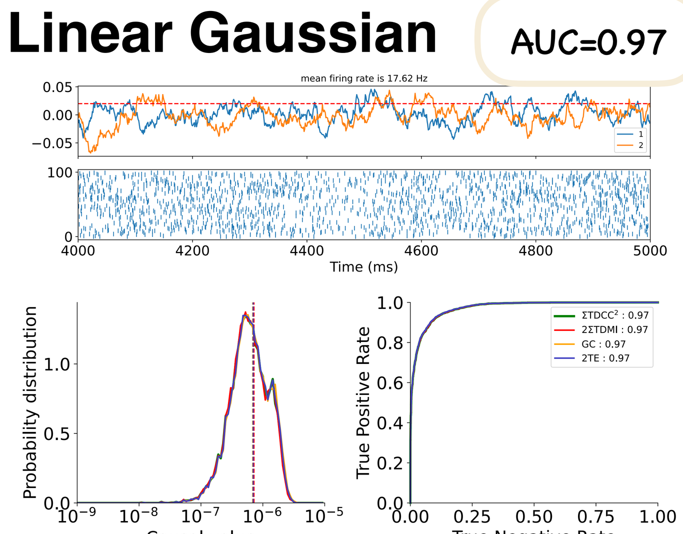
E-I HH networks

AUC=0.99



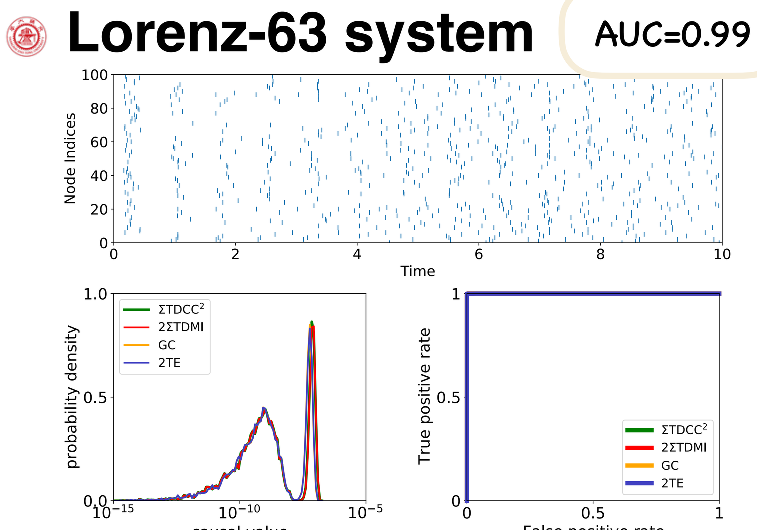
Linear Gaussian

AUC=0.97

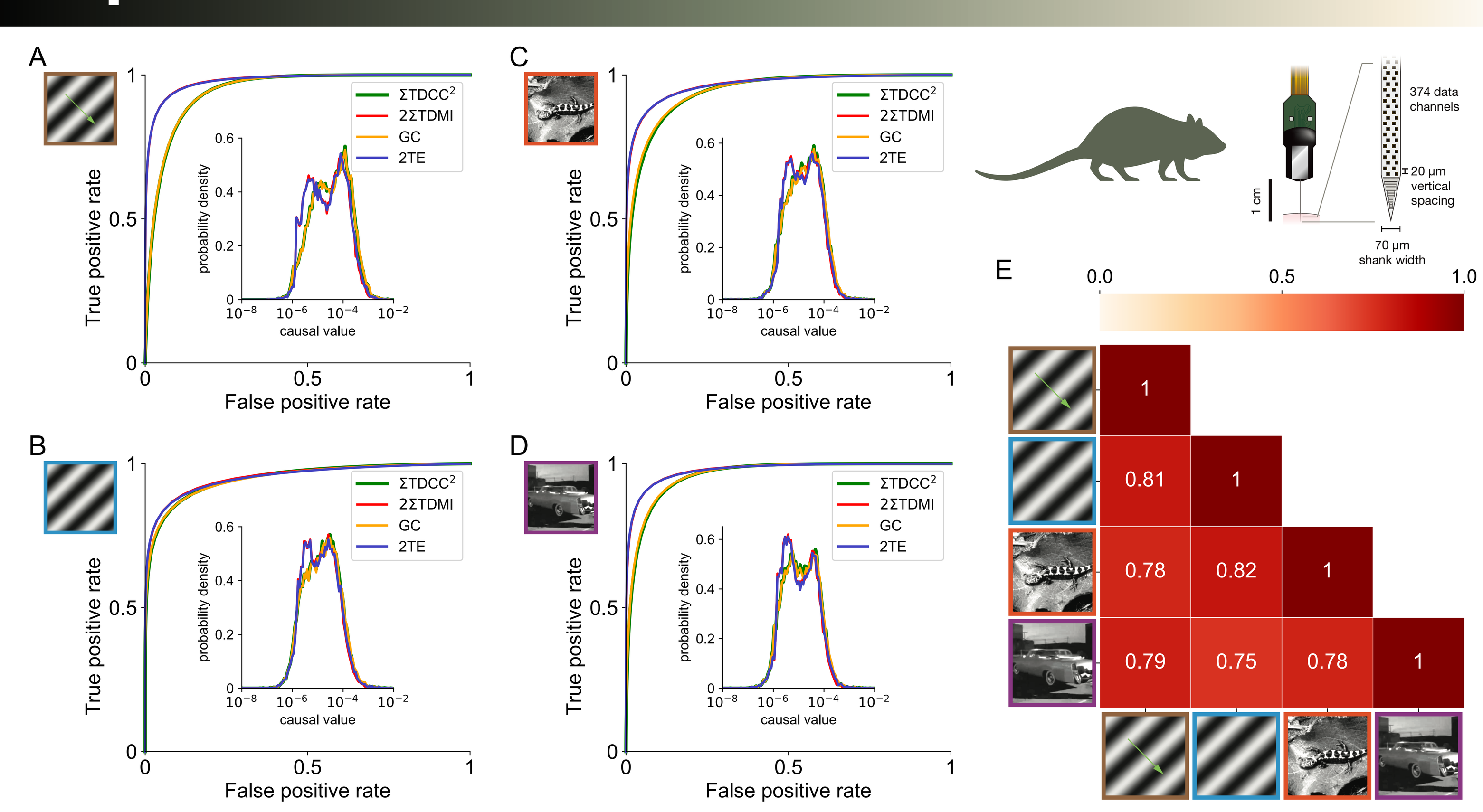


Lorenz-63 system

AUC=0.99



Experimental Data



- Mathematical relations hold in the case of Neuralpixel spike train data.
- Reconstruction with high AUC value and high consistency across stimuli.

Conclusions

- Theorems: quantitative relationships between four causal measures based on pulse-output signals.
- Accurate reconstruction of connectivity in HH networks.
- Overcome the confounder issue and curse of dimensionality.
- Applicable to more general dynamical regimes.
- Reconstruction of generic network systems, including Gaussian linear models and Lorenz system.
- For Neuralpixel data, reconstructions valid and consistent.

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